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THE INFLUENCE OF SOUND ON HEAT TRANSFER FROM A CYLINDER IN CROSSFLOW

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Abstract—This paper reports the results of an experimental investigation of the influence of intense acoustic vibrations on the rate of heat transfer from a circular cylinder (¾ inch diameter) to air in crossflow. The direction of vibration was normal both to the axis of the cylinder and to the crossflow. Plane stationary sound waves were employed and the cylinder was positioned so that its longitudinal axis coincided with displacement antinodes of the sound waves; the sound frequencies used were 1100 cps and 1500 cps.

The results show that intense sound causes the overall convective heat-transfer coefficient to increase appreciably (up to 25 per cent) in two regions. In one of these regions, which corresponds to the lowest values of the crossflow Reynolds number employed in the experiments [$(Re)_{cf} \sim 1000$], the increase in the rate of heat transfer appears to be the result of an interaction similar to thermoacoustic streaming. In the second region, which corresponds to the highest values of the crossflow Reynolds number employed in the experiments [$(Re)_{cf} \sim 10\,000$], the increase in heat transfer appears to be the result of two different interactions: (1) a resonance interaction between the acoustic oscillations and the vortices shed from the cylinder; (2) a modification of the flow in the laminar boundary layer on the upstream portion of the cylinder similar to the effect of free stream turbulence. Local heat-transfer data are presented which support this hypothesis. Empirical equations are developed by means of which it is possible to calculate the maximum increase in the Nusselt number caused by a given sound wave in the second region [$(Re)_{cf} \sim 10\,000$].

NOMENCLATURE

a, sinusoidal displacement amplitude of vibration, ft;
b, B, constants;
D_o, diameter of test cylinder, ft or in;
f, frequency of vibration, cps;
F, weighting factor, [see equation (3)];
g, gravitational constant, 32·2 ft/s²;
Gr, Grashof number (based on cylinder diameter);
h, heat-transfer coefficient, Btu/h ft² degF;
K_b, dimensionless parameter; $K_b = P/\rho_a U^2$ [see equation (5)];
K_c, dimensionless parameter; $K_c = P/l\, \rho_a\, \beta\, \Delta t$ [see equation (1)];
l, half wavelength of sound, ft;
L, length of test section of cylinder, in;
L_x, integral length scale of turbulence;
n, constant;
Nu, Nusselt number (based on cylinder diameter);
P, mean acoustic radiation pressure, lbf/ft²;
Pr, Prandtl number;
$\bar{Q}$, total heat dissipation from test section, W;
$(Re)_{cf}$, crossflow Reynolds number (based on U and cylinder diameter);
$(Re)_v$, vibrational Reynolds number (based on u and cylinder diameter),

$$(Re)_v = \frac{u}{U}(Re)_{cf} = \epsilon\,(Re)_{cf};$$

SPL, sound pressure level, dB, re 0·0002 μb;
t_a, ambient temperature, °F;
t_s, surface temperature, °F;
t_f, mean film temperature, °F; $t_f = (t_a + t_s)/2$;
u, root-mean-square velocity of vibration or turbulence, ft/s;
U, crossflow velocity, ft/s;

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v, velocity amplitude of vibration, ft/s; $v = 2\pi af$.

Greek symbols

β, coefficient of volumetric expansion of air at constant pressure, (degF abs.)$^{-1}$;

δ_{ac}, AC boundary-layer thickness, ft; $\delta_{ac} = \sqrt{\frac{\nu}{\omega}}$;

Δt, temperature difference, degF; $\Delta t = (t_s - t_a)$;

ϵ, level of vibrations; $\epsilon = \frac{u}{U}$;

λ, wavelength of sound, ft; $\lambda = 2l$;

ν, kinematic viscosity of air; ft^2/s;

ω, circular frequency, rad/s; $\omega = 2\pi f$;

ϕ, function defined by equations (6) and (8);

ψ, function defined by equations (6) and (8).

Subscripts

a, indicates ambient conditions; thus t_a = temperature of ambient fluid;

o, indicates absence of vibrations; thus $(Nu)_o$ = Nusselt number in absence of vibrations;

v, indicates presence of vibrations; thus $(Nu)_v$ = Nusselt number in presence of vibrations.

Superscript

*, indicates local maximum values of $(Nu)_v/(Nu)_o$ [see Fig. 10].

INTRODUCTION

THE influence of vibrations upon the rate of heat transfer from a heated surface can be investigated experimentally by two different methods. In the first method, the surface is held stationary and acoustic vibrations are established in the fluid medium surrounding the surface. In the second method, an oscillatory motion is impressed upon the surface itself—this method generally employs a reciprocating connecting rod and crank mechanism or a mechanical system which is caused to vibrate at its resonant frequency. Both the acoustical, or "sound method", and the "mechanical vibrations method" have the same basic objective: to create an oscillating *relative velocity* vector between a heated surface and a fluid medium. The two methods are directly comparable if the characteristic dimensions of the test object are simultaneously large compared to the amplitude of vibration and small compared to the wavelength of sound. When these conditions are satisfied, the decision as to which method shall be used in a particular situation depends primarily upon the region of interest in the frequency of vibration. Generally speaking, the mechanical method is more convenient for investigations conducted at low frequencies (less than approximately 500 cps), whereas the sound method is usually more suitable for studies conducted at higher frequencies.

The interaction between vibrations and heat transfer has been studied for several different geometries of heat-transfer surfaces and for different orientations of the vibration vector relative to these surfaces; however, for practical reasons, the geometry which has been most intensively studied is that of a circular cylinder subjected to vibrations whose direction is normal (transverse) to the longitudinal axis of the cylinder. Until quite recently, the majority of research in this area of study has dealt with natural convection (as opposed to forced convection); this limitation was a consequence of the desire on the part of research workers to reduce to a minimum the number of primary variables which would have to be considered during the initial studies of this complex problem. The initial studies demonstrated that the influence of vibrations on heat transfer can be large; this potentially useful discovery has aroused considerable interest and, as a consequence, the research effort in this area has expanded—it currently includes studies in both natural and forced convection for gases and also for liquids.

The "state of the art" in this field of study has not yet progressed to the point where broad generalizations over wide ranges of the controlling parameters can be made, for the problem is very complex and the available data refer to specific conditions which are difficult to relate. The present work was undertaken in order to obtain data on the influence of high intensity vibrations on forced convection from a cylinder whose diameter is large compared to the displacement amplitude of vibration. These data provide information under conditions which have not been systematically investigated heretofore, and they tie together the results of several earlier studies to form a more coherent picture of the overall problem.

SURVEY OF THE LITERATURE

The forced convection experiments with sound which form the basis of the present study are related to analogous natural convection investigations; for this reason, a brief résumé of the known facts concerning the interaction between vibrations and natural convection heat transfer from cylinders is given below. Also, since the results of the present work are related in some respects to the results of studies of the influence of free stream turbulence on heat transfer, a short discussion of the influence of free stream turbulence on forced convection for cylinders in crossflow is included in the survey of the literature which follows.

Natural convection and vibrations

One of the earliest investigations of the interaction between vibrations and heat transfer is that of Martinelli and Boelter [1]. They studied the effect of vertical vibrations upon the rate of heat transfer from a horizontal tube immersed in a tank of water. The tube diameter was $\frac{3}{4}$ in, the amplitude of vibration was from 0 to 0·10 in, the frequency range was from 0 to 40 cps, and the temperature difference, Δt, varied from 8 to 45 degF. It was found that at low vibration Reynolds numbers the coefficient of heat transfer was unaffected; this result was attributed to the dominance of free convection in this range. For sufficiently intense vibrations, however, the coefficient of heat transfer was observed to increase by as much as 400 per cent of its value without vibrations. No effort was made to observe the boundary-layer flow around the cylinder. The data were correlated by means of a dimensionless equation relating the Nusselt number with the Grashof, Prandtl, and vibration Reynolds numbers. In a later communication [2], Boelter reported that results obtained from subsequent experiments did not agree with the original data. Recently, Larson and London [3] have suggested that cavitation, which, they found, has a major effect on heat transfer, may have occurred in the Martinelli–Boelter experiments. The occurrence of cavitation could explain why efforts to reproduce the original Martinelli–Boelter results were unsuccessful, for it is known that the occurrence of cavitation in water is a function of the gas content of the water, and the gas content of the water in the original Martinelli–Boelter tests may have differed from the gas content in subsequent experiments.

Kubanskii has performed experimental studies [4, 5, 6] of the influence of stationary sound fields on free convection from a heated horizontal cylinder in air. In Kubanskii's experimental work, the direction of vibration was parallel to the axis of the cylinder (diameter = 2·4 cm). The results were correlated by the relation

$$(Nu)_v = 10\, K_c^{0.15} \qquad (1)$$

where $(Nu)_v$ is the Nusselt number in the presence of sound and $K_c = (P/l\rho_a\beta g\Delta t)$ is the ratio of the gradient of the mean acoustic radiation pressure, P/l, to the buoyant force per unit volume of fluid, $\rho_a\beta g\Delta t$; in (1) the physical quantities in $(Nu)_v$ and K_c are evaluated at the mean film temperature. It is worthwhile to note that since (1) does not collapse to the natural convection heat-transfer equation in the absence of sound (that is, when $P \to 0$), there must be a lower bound on K_c below which (1) is not valid. A plot of the data from which (1) was obtained [7] shows that the lowest value for K_c utilized in the experiments was 1·6; pending further information, $K_c = 1{\cdot}6$ may be taken as the least value for which (1) is applicable.

Kubanskii also performed a theoretical analysis [7] of the sound and heat-transfer interaction to which the empirically determined equation (1) applies; this analysis was based on the assumption that the influence of natural convection and sound upon the Nusselt number are linearly additive. The calculated results of the analysis are in reasonably good agreement with the experimental data. The underlying physical reason why the assumption of linear addition is permissible in Kubanskii's analysis is that, in this particular case, the effect of sound is merely to distort the natural convection flow field by acoustic streaming.† Kubanskii verified this assumption experimentally by a visual technique, which clearly showed that the coupled flow corresponded to the pattern one would expect from the superposition of acoustic streaming upon natural convection. This kind of correspondence does not always occur, for it has been shown [9] that the interaction of natural convection and sound can result in entirely new patterns of flow, which cannot be predicted by simple superposition; in such cases, the assumption of linear addition is not valid [10].

The influence of vibrations upon the rate of heat transfer from horizontal heated wires has been investigated by Lemlich [2]. In these experiments, electrically heated wires of three different sizes (0·0253 in, 0·0396 in and 0·0810 in) were vibrated with sinusoidal amplitudes up to 0·115 inches in the frequency range from 39 to 122 cps. The range of the temperature difference between the wires and the ambient air was 7–365 degF. The coefficients of heat transfer for vibrating wires were as much as four times the coefficients without vibrations but with other conditions unaltered. For increases in heat transfer in excess of 10 per cent, Lemlich correlated his experimental data by the following empirical equation:

$$\frac{h_v}{h_o} = 0{\cdot}75 + 0{\cdot}0031\,\frac{\overline{Re}^{2\cdot05}(\beta\Delta t)^{0\cdot33}}{(GrPr)^{0\cdot41}} \qquad (2)$$

where h_v is the heat-transfer coefficient with vibration, and h_o is the analogous coefficient without vibration; the Reynolds number, $\overline{Re}$, in the above equation was computed on the basis of the cylinder diameter and the average vibrational velocity, $\bar{v} = 4af$.

Lemlich reported similar results for both vertical and horizontal vibrations. In order to account for this observation, the concept of a "stretched film" surrounding the entire path of vibrations was proposed. An effort was made to observe the boundary-layer flow using smoke, by holding a lighted cigarette under the heated wires, but these smoke observations were inconclusive. Kubanskii [5] has rejected the stretched film hypothesis and has suggested, instead, that the true cause of the increase in heat transfer in Lemlich's experiments was acoustic streaming near the wires. In view of the uncertainty concerning the character of the flow in Lemlich's experiments, Kubanskii's categorical statement must be regarded with reservation. To support his statement, Kubanskii cited the work of West [11] who showed that acoustic streaming occurs near a small vibrating pin. However, in West's work the amplitude of vibration was approximately half the diameter of the vibrating pin; whereas, in Lemlich's work the amplitude of vibration was as much as four or five times the diameter of the wires. Now, since it has been shown [8] that the orderly flow which is generally called acoustic streaming breaks down when the amplitude-to-diameter ratio is greater than about 0·5, it appears that the term acoustic streaming, as it is usually defined, cannot be used to describe the interaction which occurred in Lemlich's experiments. It is suggested that the flow near the wires in Lemlich's high amplitude-to-diameter tests may properly be described as "unsteady forced convection"; this description does not conflict, in essence, with Lemlich's stretched film hypothesis, but it describes more clearly the kind of interaction which occurs under these conditions. This viewpoint is supported by the results of a recent experimental study of heat transfer from an oscillating horizontal wire to water by Deaver, Penny and Jefferson [12]. In this study, it is shown that for large-amplitude vibrations (~1 in) the heat-transfer rate from a horizontal wire (0·007 inches diameter) to water can be correlated by an equation which is almost identical to the equation for forced convection.

Fand and Kaye [13] have performed a quantitative experimental investigation of the influence of transverse sound fields upon free convection from a horizontal cylinder in air; the diameter of the cylinder was $\frac{3}{4}$ in, and the vibration vector was horizontal. The data showed that a "critical sound pressure level" exists below which the influence of sound is negligible and above which the rate is markedly increased by sound (critical SPL = 140 dB; af 0·36 ft/s). The data also showed that for sound waves whose half-wavelength was six or more times the

† Acoustic streaming refers to the time-independent components of flow induced by oscillations in a fluid [8].

diameter of the test cylinder ($l/D_o \geqslant 6$), the coefficient of heat transfer is a function of only two variables; namely, the temperature difference, Δt, and the intensity of vibration, defined as the product of amplitude and frequency, af.

In order to gain insight into the physical mechanism which caused the rate of heat transfer to increase in the presence of sound, Fand and Kaye [9] performed a flow-visualization study, using smoke as the indicating medium. This study revealed that at the critical sound pressure level the typical free-convection boundary-layer flow pattern around the heated cylinder is disrupted, and a fundamentally different type of boundary-layer flow, called thermoacoustic streaming, develops. Thermoacoustic streaming is characterized by a pair of oscillating vortices which begin to appear above the upper surface of the cylinder when the sound pressure level reaches the critical value. As the sound pressure level is increased beyond the critical value, the vortices eventually reach a stage of development wherein their character is fully established. When this stage has been reached, a further rise in the sound pressure level increases the size of the vortices but does not alter their form; at this stage the fluid pattern resembles vortex shedding behind a cylinder in forced flow normal to its axis (cross-flow). Thermoacoustic streaming has also been observed by Sprott, Holman, and Durand [14].

For $l/D_o \geqslant 6$, Fand and Kaye found that the vibration intensity for fully developed thermoacoustic streaming is $af = 0{\cdot}71$ ft/s (SPL = 146 dB), and that, for $af \geqslant 0{\cdot}71$ ft/s and $\Delta t \leqslant 400$ degF, the heat-transfer coefficient may be determined from the following empirical correlation equation:

$$h_v = 0{\cdot}722\,[\Delta t(af)^2\,F]^{1/3}. \qquad (3)$$

In (3) the factor F is a geometrical weighting factor defined and evaluated in [13]; the numerical value of F approaches unity as the ratio of half-wavelength to cylinder diameter approaches infinity. It is of interest to note that the velocity amplitude, v, for a simple harmonic oscillation is given by $v = 2\pi af$, from which it follows that h_v in (3) is a function of the kinetic energy of vibration.

Fand and Peebles [15] have shown that thermoacoustic streaming occurs, and that (3) is valid, for a much wider range of frequency than was originally considered in [13]. Also, an experimental investigation of the local heat-transfer coefficient for the same conditions as pertain to the overall heat-transfer coefficient in (3) has been made [16]. The local heat-transfer results (see Fig. 1) show that superposition of intense sound upon the free-convection field about a heated horizontal cylinder increases the heat-transfer coefficient both on the under and upper portions of the cylinder; the maximum percentage increases in the local heat-transfer coefficient on the under and upper portions of the cylinder are approximately 250 and 1200 per cent, respectively.

The influence of vertical mechanical vibrations on heat transfer by free convection from a horizontal cylinder has been investigated [17]. In this experimental study the diameter of the cylinder was $\frac{7}{8}$ in and the ranges of the primary experimental variables were as follows: temperature difference Δt, 25–185 degF; amplitude of vibration a, 0–0·16 in; frequency of vibration f, 54–225 cps; intensity of vibration af, 0–1·22 ft/s. Here, too, the data show that the sole controlling vibrational variable is the intensity of vibration. For intensities of vibration less than 0·3 ft/s, the influence of vibrations upon the coefficient of heat transfer is negligible; above this "critical" intensity, the effect of vibrations is to increase the heat-transfer coefficient significantly. A flow visualization study employing smoke was performed in conjunction with these heat-transfer experiments. This smoke study indicated that the fluid-dynamical mechanism which caused the observed increases in the heat-transfer coefficient was vibrationally induced turbulence. This turbulent type of boundary-layer flow differs radically from thermoacoustic streaming. In the experimental region defined by $\Delta t \geqslant 100$ degF and $af \geqslant 0{\cdot}9$ ft/s, the following empirical correlation equation was developed for the heat-transfer coefficient:

$$h_v = 0{\cdot}847\left(\frac{\Delta t}{D_o}\right)^{0{\cdot}2}(af). \qquad (4)$$

It was suggested that the vibrational intensity $af = 0{\cdot}9$ ft/s corresponds to "fully developed

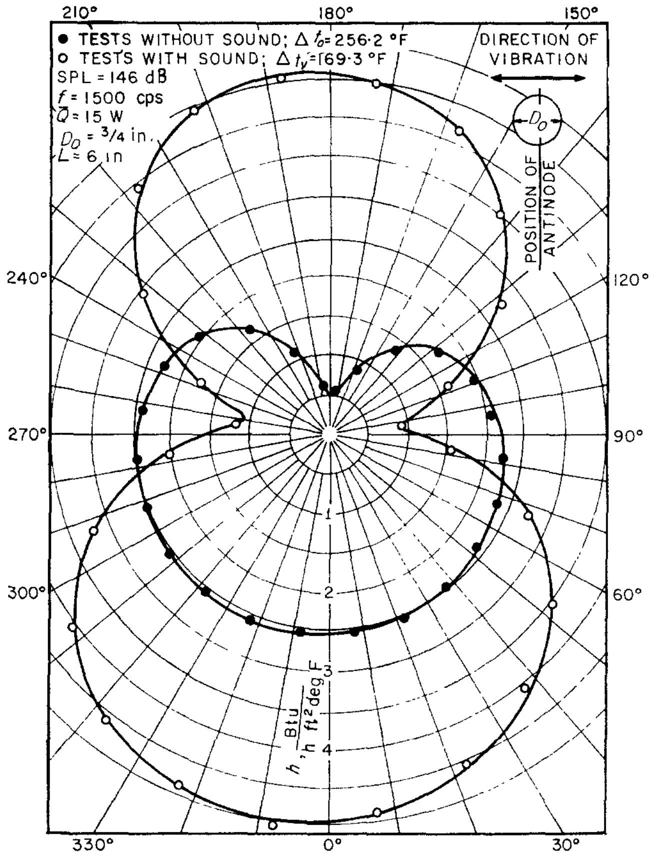


FIG. 1. Local heat-transfer coefficient around a heated horizontal cylinder, with and without sound at constant $\bar{Q}$ (from [16]).

turbulent flow" in the neighborhood of the cylinder; this terminology is analogous to the descriptive phrase "fully developed vortex flow" which had been used to describe the flow around a heated cylinder subjected to intense transverse horizontal acoustical vibrations. Since the velocity amplitude for a simple harmonic oscillation is given by $v = 2\pi af$, it follows that the heat-transfer coefficient in (4) is directly proportional to the velocity of vibration.

Forced convection and vibrations

Kubanskii [18] has made experimental studies of the influence of stationary sound waves on the boundary-layer flow near, and heat transfer from, a cylinder in crossflow for two different geometries: Case 1—for sound waves directed along the axis of the cylinder; Case 2—for sound waves perpendicular both to the axis of the cylinder and to the direction of crossflow (the geometry of the second case is similar to that of the present study, shown in Fig. 7). The boundary-layer observations were made with a cylinder 32·5 cm long by 2·4 cm in diameter; the heat-transfer studies were made with a cylinder 12 cm long by 1·5 cm in diameter. The sound field was produced by a whistle, the radiations of which were focused in a parallel beam with the aid of parabolic reflectors which were 18 cm in diameter. As a result of the collimation into a beam, the sound wave was restricted to a relatively small volume in space and had an uneven intensity along its front (the ends of the 32·5 cm long cylinder projected outside the sound field). The intensity at the center of the beam was from 0·03 to 0·34 W/cm². The boundary-layer studies were made with $\lambda = 2{\cdot}8$ cm and $(Re)_{cf} = 2500$; the heat transfer studies were made with $\lambda = 2{\cdot}0$ cm and $\lambda = 2{\cdot}5$ cm, and with $(Re)_{cf}$ restricted to the narrow range from 1450 to 1770.

Kubanskii observed that his stationary sound fields produced marked changes in the boundary-layer flows near the cylinders, provided that the acoustic particle velocity was equal to or greater than the crossflow velocity. When this condition on the intensity was satisfied in Case 1, the boundary layer was periodically distorted in space along the length of the cylinder, and the period of distortion was $\lambda/2$; when this condition on the intensity was satisfied in Case 2, Kubanskii noticed that it was possible to move the point of separation either upstream or downstream, depending upon the location of the nodes and antinodes of the stationary wave relative to the axis of the cylinder. Kubanskii stated that all these boundary-layer effects could be explained by the simple superposition of acoustic streaming, which, for these geometries, consists of a steady flow from the antinodes to the nodes of the standing wave, upon the convective flow field near the surface of the cylinder.

As stated earlier, Kubanskii's heat-transfer measurements were taken with a cylinder 12 cm long and 1·5 cm in diameter. This cylinder could be placed wholly within the acoustical field, but because of the nonuniformity of the field, the middle portion of the cylinder was located at a zone of significantly greater sound intensity than were the ends; this being so, great care must be exercised in attempting to apply Kubanskii's quantitative heat-transfer results to situations in which the distribution of sound intensity differs appreciably from the prototype. In order to achieve marked changes in heat emission with sound, Kubanskii kept his crossflow velocity low, in the range from 1·45 to 1·77 m/s (this velocity range corresponded to a Reynolds number range from 1450 to 1770). In Case 1, Kubanskii observed that the effect of sound was always to increase the heat-transfer rate; but in Case 2 he reported that he was able to either increase or decrease the convective heat-transfer rate by moving the cylinder relative to the nodes of the stationary sound waves. Kubanskii correlated the increases in heat transfer which he measured by means of equations of the following form:

$$(Nu)_v = B(K_b)^n \tag{5}$$

where $K_b = P/\rho_a U^2$ is the ratio of the mean acoustic radiation pressure to the dynamic head; the fluid properties in (5) were evaluated at the temperature of the stream far from the cylinder. The values of B and n in (5) are as follows:

$B = 28$, $n = 0{\cdot}08$ for $u < U$ —for Case 1 (from [19])

$B = 23$, $n = 0{\cdot}06$ for $u < U$, $B = 27$, $n = 0{\cdot}2$ for $u > U$ —for Case 2 (from [18]).

The quantity u is the root-mean-square of the vibrational velocity, and U is the crossflow velocity. Kubanskii hypothesized that the break in the curve for $(Nu)_v$ when $u = U$ in Case 2 is caused by a transition from orderly acoustic streaming to turbulence. (The values for B and n in Case 1 for $u > U$ were not given.)

In a recent publication [19], Kubanskii attempted to provide a theoretical basis for his experimental forced - convection - plus - sound results. This analysis was based on the same fundamental assumption as was made by Kubanskii in his analysis of free convection; namely, that acoustic streaming and convective effects are linearly additive. Several additional assumptions were made in the analysis, including the assumption that the average temperature gradient at the cylinder wall may be replaced by the average temperature gradient at the outer limit of the laminar sublayer. Kubanskii's experimental and analytical results for Cases 1 and 2 are shown in Figs. 2 and 3. These figures show that there is considerable scatter in the experimental data and also, surprisingly, in the theoretical results; no explanation was given by Kubanskii for the apparent inconsistencies in the theoretical calculations.

The effect of vibrations on heat transfer from a cylinder in crossflow has also been investigated by Sreenivasan and Ramachandran [20]; here, too, the vibration vector was perpendicular both to the cylinder axis and to the direction of

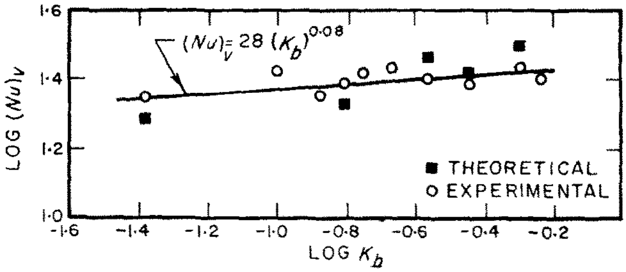


FIG. 2. Kubanskii's results for Case 1: Sound waves directed along axis of cylinder.

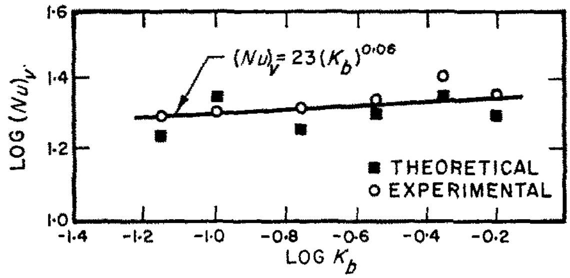


FIG. 3. Kubanskii's results for Case 2: Sound waves perpendicular to axis of cylinder and direction of crossflow.

crossflow (this geometry is similar to Kubanskii's Case 2 and to the geometry of the present study shown in Fig. 7). The diameter of the cylinder was 0·344 in, the amplitude of vibration (mechanically induced) varied from 0·15 to 0·63 in, the crossflow velocity varied from 19 to 92 ft/s [corresponding to $(Re)_{cf}$ from 2500 to 15 000], and the frequency of vibration was from approximately 3 to 45 cps. No appreciable change in the heat-transfer coefficient was observed by imposing root-mean-square vibrational velocities as high as 20 per cent of the crossflow velocity. This insensitivity of the heat-transfer coefficient to vibration was shown to agree with an analysis based on the resultant of the vibrational and crossflow velocities.

A theoretical analysis of the laminar boundary layer on a hot cylinder in a fluctuating stream has been performed recently by Gribben [21]. The external flow considered was that of a steady basic velocity with a superimposed small-amplitude oscillation directed parallel to the main stream (this geometry differs from the geometry employed by Kubanskii, Sreenivasan, and the present authors). Gribben obtained approximate solutions of the equations for nonsteady, two-dimensional low-speed compressible flow by the Pohlhausen method; the assumption of quartic profiles for velocity and temperature was made. Expressions for the steady and unsteady components of the skin friction and stagnation-point heat transfer were derived. The accuracy of these approximate results cannot be determined at the present time since no comparable exact solutions or experimental data are known.

Forced convection and free stream turbulence

Much effort has been expended on the study of the influence of free stream turbulence on heat transfer. These studies have shown that convective heat-transfer rates generally increase with an increase in turbulence [22, 23, 24, 25]. The increase may be due to one or more of the following effects: modifications of the flow in laminar boundary layers; earlier transition from laminar to turbulent boundary-layer flow; and movement of the point of flow-separation, with attendant alteration of the flow in the wake. Giedt [22] was the first to demonstrate these separate effects, by measuring local heat-transfer rates on a cylinder in crossflow. In Giedt's experiments the crossflow Reynolds number varied from 95 000 to 213 000 and the intensity of turbulence (grid-induced) was estimated to vary from about 1 to 4 per cent. Some of Giedt's results are shown in Fig. 4.

The local heat-transfer data in Fig. 4 indicate that the heat-transfer rate in the laminar region of flow on the front half of the cylinder was increased appreciably by turbulence both for

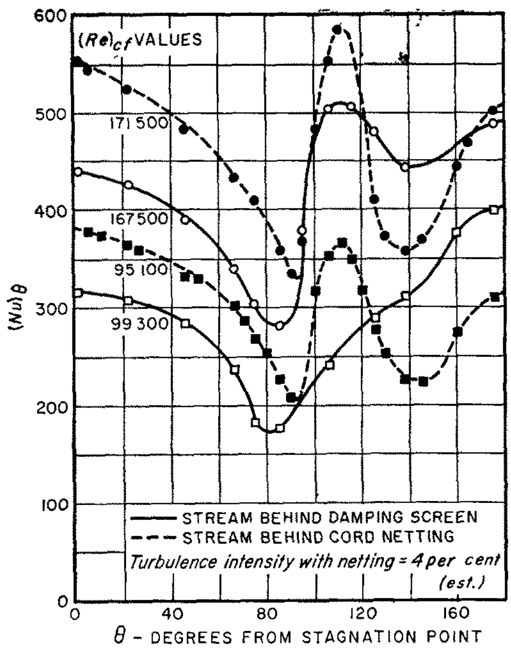


FIG. 4. Local heat transfer around a cylinder for different crossflow Reynolds numbers (after Giedt [21]).

$(Re)_{cf} \cong 95\,000$ and for $(Re)_{cf} \cong 170\,000$. The figure also shows that grid-induced turbulence caused major changes in the local heat-transfer rate on the back half of the cylinder for $(Re)_{cf} \cong 95\,000$, but less so for $(Re)_{cf} \cong 170\,000$. The reason for these differences in behavior is that for $(Re)_{cf} \cong 95\,000$ the grid-induced turbulence caused transition from laminar to turbulent flow, as is demonstrated by the appearance of a local maximum in the heat-transfer coefficient at $\theta = 110°$; whereas, for $(Re)_{cf} \cong 170\,000$, transition to turbulent flow had already begun, and the insertion of a turbulence-generating grid merely completed the transition process, thereby causing a relatively smaller change in the velocity distribution on the back half of the cylinder. Giedt's measurements showed that grid-induced turbulence (intensity ~4 per cent) caused the average heat-transfer coefficient on a cylinder to increase between 10 and 20 per cent; however, the average heat-transfer coefficient does not indicate the relative magnitudes of the changes on the front and back halves of the cylinder, for the overall change is due to an increase of heat transfer on the front half and a decrease on the back half.

Comings, Clapp and Taylor [26] have investigated the effect of turbulence on the overall heat-transfer rate from a cylinder in crossflow. In this study, the intensity of turbulence was varied from 1 to 18 per cent by inserting different grids in the stream; the range of the crossflow Reynolds number was from 400 to 20 000. It was found that increased turbulence at constant Reynolds number caused a maximum of 25 per cent increase in the overall rate of heat transfer. However, in the ranges tested, it was found that the heat-transfer rate was augmented by increased turbulence at higher Reynolds numbers [$(Re)_{cf} \sim 5000$], but this effect was negligible at lower Reynolds numbers [$(Re)_{cf} \sim 500$]; also, for $(Re)_{cf} \sim 5000$, variations in the intensity of turbulence at higher levels of turbulence (7–18 per cent) had little influence on the heat-transfer rate, whereas variations in the intensity of turbulence at lower levels (1–3 per cent) had pronounced effects.

A plausible explanation for these results is as follows: the data of Comings *et al.* were all taken well below the critical Reynolds number; however, the experimental range was sufficiently large so that the ratio of the viscous forces to inertia forces in the boundary layer for the lower values of Reynolds number [$(Re)_{cf} \sim 500$] was an order of magnitude larger than for the higher values of the Reynolds number [$(Re)_{cf} \sim 5000$]. Therefore, within the subcritical region in which Comings *et al.* worked, the boundary layers at the lower values of the Reynolds number were considerably more stable, that is, less susceptible to transition due to free stream turbulence than at the higher values; also, with lower Reynolds numbers and relatively more viscous and thicker laminar boundary layers, free stream turbulence was more effectively damped out, and was, therefore, less effective in modifying the velocity and temperature profiles near the surface. Thus, the insensitivity of the heat-transfer rate to changes in the intensity of free stream turbulence in the experiments of Comings *et al.* for $(Re)_{cf} \sim 500$ can be explained by the hypothesis that, for this range of $(Re)_{cf}$, the laminar boundary layer on the cylinder was sufficiently stable to prevent transition and sufficiently viscous and thick to prevent appreciable modifications of the velocity and temperature profiles; for $(Re)_{cf} \sim 5000$ the converse argument can, of course, be cited. Once transition occurs, further increases in the intensity of free stream turbulence can cause relatively minor changes in the flow; this could explain why Comings *et al.* found that variations in the intensity of turbulence at their higher levels of turbulence (7–18 per cent) had less influence on the heat-transfer rate than similar variations at their lower levels of turbulence (1–3 per cent) when $(Re)_{cf} \sim 5000$.

Kestin and Maeder [24] have investigated the influence of screen-induced turbulence (intensity: 0·58–2·68 per cent) on heat transfer from a cylinder in crossflow in the range of Reynolds numbers from 128 000 to 308 000. As in the work of Comings *et al.*, it was observed by Kestin and Maeder that changes in the intensity of turbulence at lower levels of turbulence (in this case 0·6–1·2 per cent) were more effective in altering the heat-transfer rate than were equivalent changes in intensity at higher levels of turbulence (1·4–2·7 per cent). Thus, it appears that the hypotheses proposed above to explain the results of Comings *et al.* also apply to the results obtained by Kestin and Maeder; it is, however, important to note that Kestin and Maeder's experiments were carried out at much higher Reynolds numbers than were those of Comings *et al.*, and consequently the intensity of turbulence required to achieve transition in Kestin and Maeder's experiments is lower than in the case of Comings *et al.*

Van der Hegge Zijnen [27] has studied heat transfer from vibrating wires and from wires and cylinders to crossflows containing various intensities and scales of grid-induced turbulence. The crossflow Reynolds numbers ranged from 60 to 25 800; the intensity of turbulence from 2 to 13 per cent; and the ratio between the integral scale of turbulence and cylinder diameter varied from 0·31 to 240. The results show that the average rate of heat transfer from vibrating fine wires to a crossflow is very nearly the same as the heat transfer from a stationary wire to the same crossflow, and that the heat transfer from wires to a turbulent crossflow with a comparatively large scale of turbulence is very nearly the same as the heat transfer to a laminar crossflow having the same mean velocity (in the experiments with wires the ratio of the scale of turbulence to wire diameter, L_x/D_o, ranged from 150 to 240). When the diameter of the heated cylinder was not small compared to the scale of turbulence (L_x/D_o from approximately 1 to 10), the effect of turbulence was to increase the heat-transfer rate appreciably; for example, for $L_x/D_o = 1$ and turbulence intensity equal to 10 per cent, the convective heat-transfer rate exceeded the corresponding heat-transfer rate in the absence of turbulence by 38 per cent.

On the basis of quite extensive test data, Van der Hegge Zijnen determined empirically a formula and a pair of functions, one involving the intensity, $\phi\,[u(Re)_{cf}/U]$, and the other the scale of turbulence, $\psi(L_x/D_o)$, by means of which the ratio of the heat-transfer rate with turbulence to the heat-transfer rate without turbulence for the same mean crossflow velocity could be computed. Thus,

$$\frac{(Nu)\text{ with turbulence}}{(Nu)\text{ without turbulence}} = \left\{1 + \phi\left[\frac{u}{U}(Re)_{cf}\right]\psi(L_x/D_o)\right\}. \quad (6)$$

Curves for determining values of ϕ and ψ are given in Figs. 5 and 6 (solid lines). One very interesting aspect of these curves is that the empirical function involving the scale of turbulence, ψ, has an "optimum" value; that is, for a certain scale-to-diameter ratio, namely, $L_x/D_o = 1{\cdot}6$, the influence of turbulence on the heat-transfer rate is a maximum. Van der Hegge Zijnen suggested that the optimum value of L_x/D_o corresponds to a condition of resonance wherein some "effective" frequency of turbulence coincides with the frequency of the eddies shed by the cylinder (Strouhal effect); apparently, this resonance reinforces the oscillations of the eddies in the wake and causes the observed maximum in the rate of heat transfer. It will be shown that the results of the present investigation tend to support Van der Hegge Zijnen's resonance hypothesis.

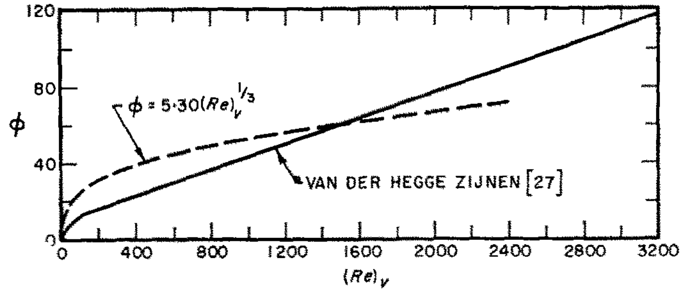


FIG. 5. ϕ as a function of $(Re)_v$.

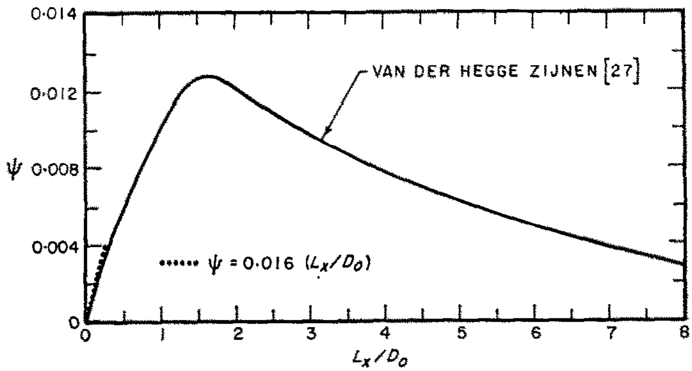


FIG. 6. ψ as a function of L_x/D_o.

APPARATUS AND PROCEDURE

Two electrically heated cylinders were used in this investigation: one for overall heat-transfer measurements, and one for local heat-transfer measurements. The diameters of the cylinders, D_o, were both $\frac{3}{4}$ in, and the length of the test sections, L, was 6 inches in both cases; the construction and methods of operating these test cylinders are described in detail in [13] and [16]. The system used to generate and measure intense stationary sound fields is also described in [13]. The experiments were performed in a specially constructed "anechoic wind tunnel," which consisted of an anechoic chamber through which a stream of air could be made to flow; the air moved upward through the floor of the chamber, past a series of fine-mesh screens, and out through the ceiling. The purpose of the screens was to reduce the level of free stream turbulence and to provide a stream with a more uniform velocity profile than would have been otherwise obtained. Throughout the investigation, the temperature of the stream was maintained at $70 \pm 5°F$ by means of an air conditioner. The basic test geometry is shown in Fig. 7, and a sketch showing the arrangement of the apparatus is given in Fig. 8.

The velocity of the air stream (crossflow velocity) was controlled by a variable speed blower and was measured with a thermal

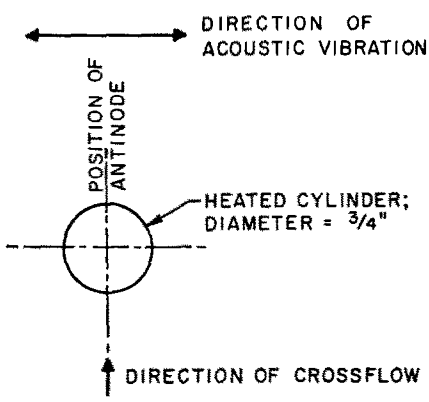


FIG. 7. Test geometry.

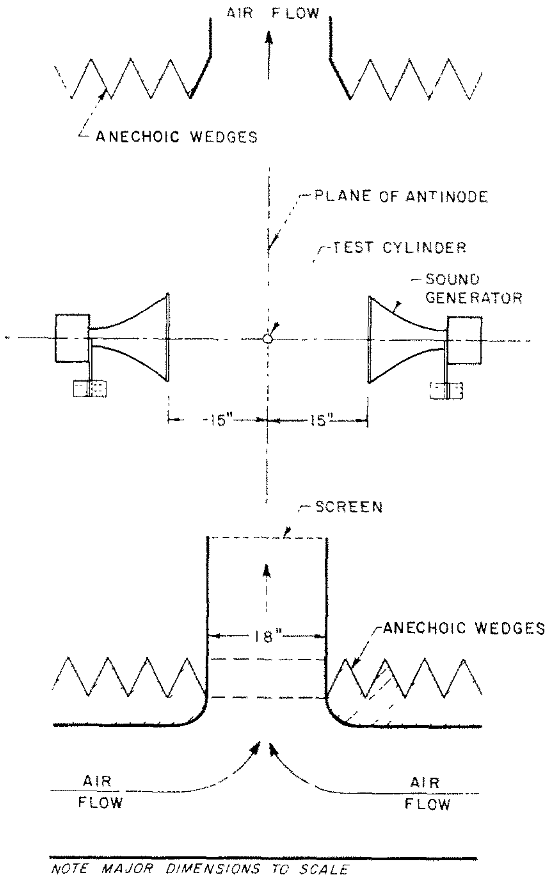


FIG. 8. Apparatus.

anemometer whose calibration was accurate to ± 3 per cent. A pitot tube mounted in the duct near the outlet of the blower (not shown in Fig. 8) was used to check the consistency of the anemometer readings. The velocity of the air stream in the vicinity of the test cylinders was explored for different blower speeds and was found to be essentially uniform in an 8-in cube centered on the test section (not including the region of the boundary layer and wake); thus, the test sections of the cylinders (6 in long) were well within the zone of uniform velocity. Crossflow velocity readings were made in the absence of sound, and sound readings were made in the absence of flow.

In order to avoid the introduction of undesirable disturbances in the flow near the test cylinder, a calibration curve of crossflow velocity versus blower rpm was made (blower rpm was measured by an electric tachometer); with the aid of this curve, it was possible to adjust the crossflow velocity to a predetermined value by appropriately adjusting and monitoring the blower rpm. This procedure permitted the removal of the anemometer from the field of flow during tests. (The readings of the pitot tube, which was mounted near the outlet of the blower, provided a running check on the blower rpm versus crossflow velocity calibration curve.) The sound measuring microphone was eliminated from the flow field in a similar way [13]; thus, the flow field near the heat-transfer cylinder was entirely unobstructed during tests.

The experimental program consisted of a series of ninety-eight overall heat-transfer tests without sound, 426 tests with sound, and four sets of local heat-transfer tests (two sets without sound, and two sets with sound). Each set of local heat-transfer tests consisted of twenty-four determinations of the local heat-transfer coefficient, taken at 15° intervals around the circumference of the cylinder. The ranges of the experimental variables were as follows: crossflow velocity, 1·5–30 ft/s [$(Re)_{cf}$, 590–10 750]; temperature difference, 50–360 degF; sound pressure level, 130–150 dB (u, 0·5–5·0 ft/s); sound frequency, 1100 cps and 1500 cps. The ratio, $\epsilon = u/U$, called the "level of vibrations", varied between 1·6 and 250 per cent; the quantity ϵ is analogous to "intensity" in the study of turbulence. The values of u corresponding to the experimental values of SPL utilized in this study are listed in Table 1.

RESULTS

Crossflow without acoustical vibrations

The results of the overall heat-transfer tests for crossflow in the absence of sound are shown in Fig. 9. This plot shows the dependence of Δt_o upon $(Re)_{cf}$ for various rates of total heat dissipation, $\bar{Q}$, from the overall heat-transfer test cylinder. This data can be correlated by Hilpert's well known equation [28]:

$$(Nu)_o = B[(Re)_{cf}]^n \quad (7)$$

where B and n depend upon $(Re)_{cf}$; the fluid properties in (7) are evaluated at the mean film

Table 1. u at antinodes versus SPL *at nodes of stationary sound fields* (t_a = 70°F); SPL = 20 log u + 136

SPL (dB)	u (ft/s)
130	0·498
132	0·627
134	0·801
136	1·01
138	1·27
140	1·59
142	2·01
144	2·53
146	3·18
148	4·01
150	5·05

temperature. The pertinent values of B and n are tabulated in Fig. 9. The mean deviation between the ninety-eight experimentally measured values of $(Nu)_o$ and corresponding values calculated from (7) is less than 3 per cent. (Corrections for radiation losses were made throughout this investigation.)

Crossflow with acoustic vibrations

The results of the overall heat-transfer tests made with crossflow in the presence of sound are presented in convenient form in Figs. 10–16. In these figures the ratio $(Nu)_v/(Nu)_o$ is plotted as a function of $(Re)_{cf}$ at constant values of SPL for different values of f and $\bar{Q}$ (fluid properties evaluated at t_f). The ratio $(Nu)_v/(Nu)_o$ is a measure of the effectiveness of sound as an agent for increasing the rate of heat transfer. The temperature difference for any data point in Figs. 10–16 may be determined from the following relationship: $\Delta t_v = \Delta t_o [(Nu)_v/(Nu)_o]^{-1}$, where Δt_o is obtained from Fig. 9 for the same $\bar{Q}$; this procedure assumes that the variation of the thermal conductivity of air is negligible for a change in the film temperature corresponding to the difference between Δt_o to Δt_v.

Figs. 10–16 show that acoustic vibrations increase the overall heat-transfer rate in two regions: the first region corresponds to the lowest experimental crossflow velocities employed [$(Re)_{cf} \sim 1000$], and the second region corresponds to the highest crossflow velocities achieved [$(Re)_{cf} \sim 10\,000$]. Between these two extremes, the curves of $(Nu)_v/(Nu)_o$ versus $(Re)_{cf}$ go through minimum values; these minima occur at $(Re)_{cf}$ approximately equal to 6000 for f = 1500 cps, and $(Re)_{cf}$ approximately equal to 4500 for f = 1100 cps. Six experimental data points for $(Re)_{cf} = 0$ (natural convection) were taken from [13] and plotted in Fig. 12; these points, together with the dashed lines in Fig. 12, indicate the behavior of $(Nu)_v/(Nu)_o$ for $(Re)_{cf} < 600$.

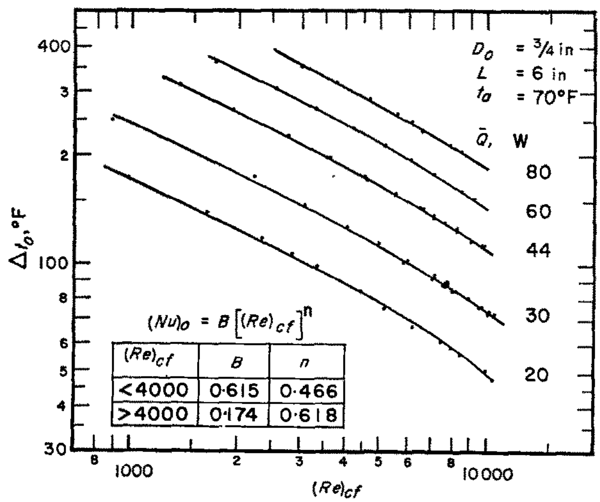
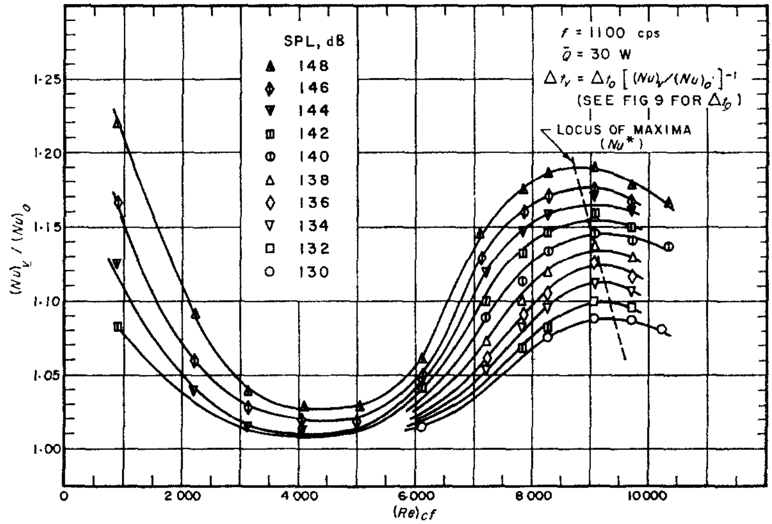


FIG. 9. Δt versus $(Re)_{cf}$ for constant $\bar{Q}$ without sound.



FIGS. 10–16. $(Nu)_v/(Nu)_o$ versus $(Re)_{cf}$ at constant sound pressure level.

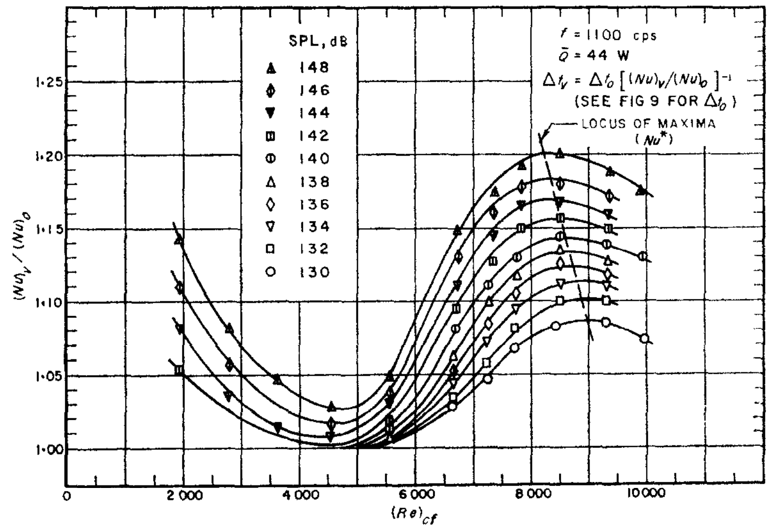


FIG. 11.

The data in Figs. 10, 11, 12, 14 and 15 demonstrate that the curves for $(Nu)_v/(Nu)_o$ possess local maxima near $(Re)_{cf} = 10\,000$; the symbol Nu^* is used to denote these local maximum values, and the symbol $(Re)^*_{cf}$ is used to denote the crossflow Reynolds number at which Nu^* occurs.

Local heat-transfer coefficient

Fig. 17 is a graph of the local heat-transfer coefficient, h, under four representative physical conditions: two of these conditions were the same as in the tests designated A_v and B_v in Fig. 17; the other two conditions, tests A_o and B_o in Fig. 17, were identical to the conditions of

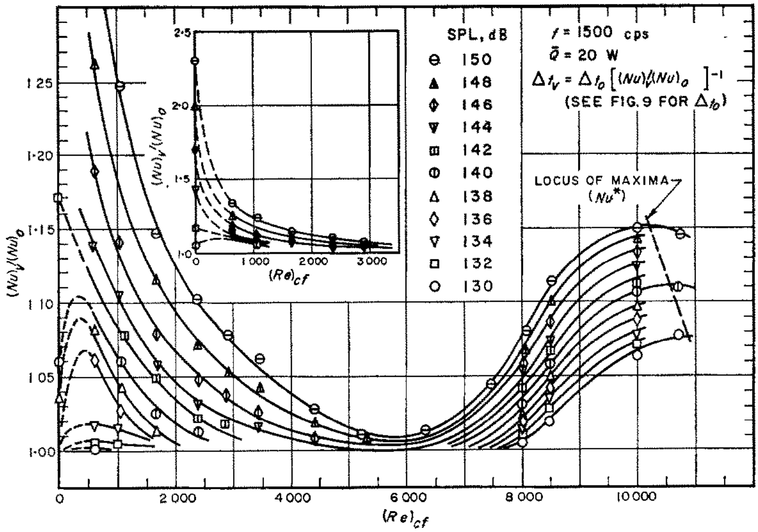


Fig. 12.

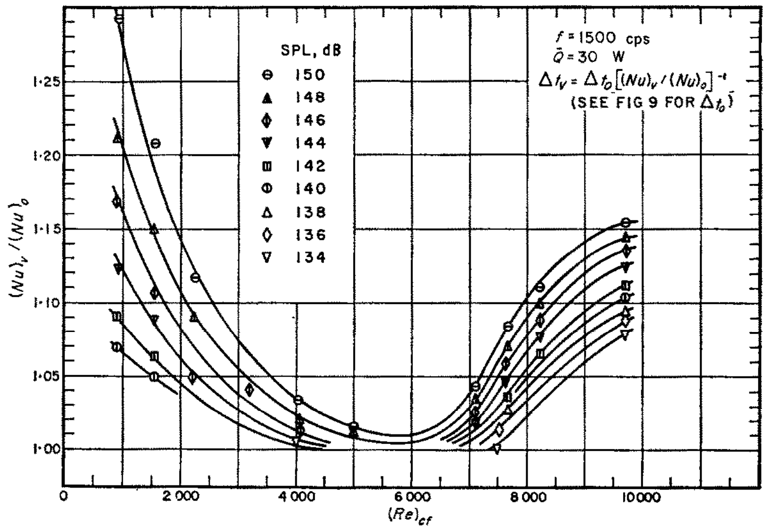


Fig. 13.

tests A_v and B_v, except that the sound field was omitted. Each point plotted in Fig. 17 is the arithmetic mean of two experimentally determined values of h which are symmetrically located with respect to the vertical center line of the test cylinder; this averaging process was performed in order to reduce errors resulting from small random variations in Δt and U.

All four conditions in Fig. 17 are marked "$\bar{Q} = 44$ W"; this signifies that all four local heat-transfer curves were determined under conditions of constant $\bar{Q}$, where $\bar{Q}$ is the total heat dissipation in the corresponding *overall*

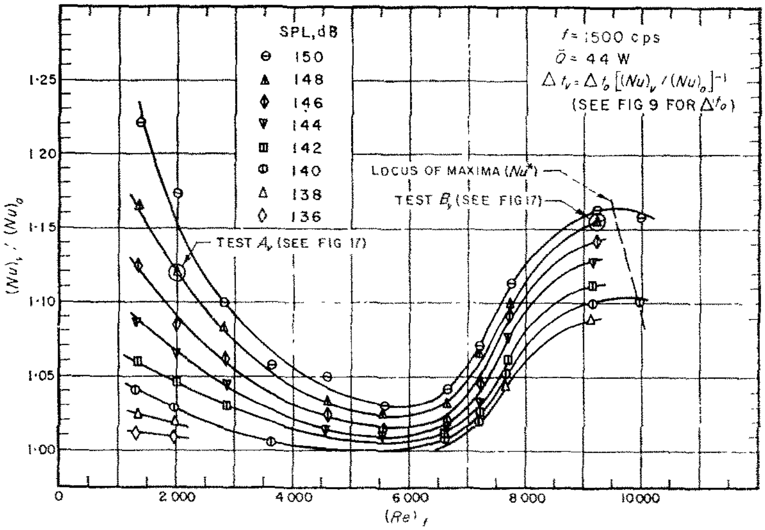


FIG. 14.

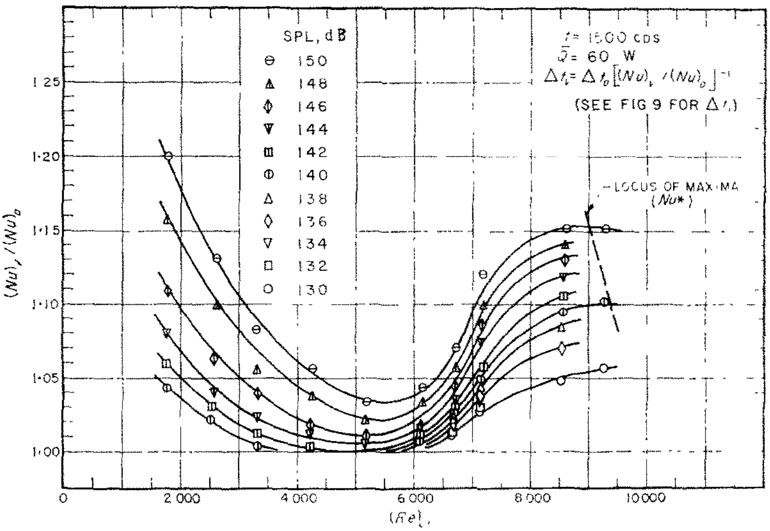


FIG. 15.

heat-transfer test. This procedure was followed in order to obtain local heat-transfer coefficients which, when integrated, could be compared directly with measured overall values for the same $\bar{Q}$. Table 2 contains a comparison of the integrated and measured overall values of the local heat-transfer coefficient for $\bar{Q} = 44$ W; the table includes comparative values of the ratio $(Nu)_v/(Nu)_o$. The entries in Table 2 show that the average heat-transfer coefficients obtained by integration of local values are 5–10 per cent lower than corresponding overall measured values; this deviation is within the limits of the experimental error [16]. The

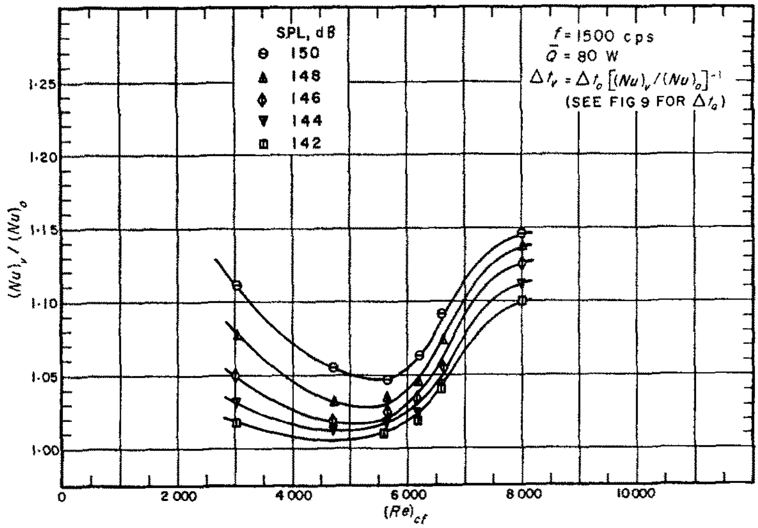


FIG. 16.

table also shows that the ratios, $(Nu)_v/(Nu)_o$, based on integrated values of the local heat-transfer coefficient, are consistently larger, approximately 5 per cent larger, than the corresponding values of $(Nu)_v/(Nu)_o$ based on overall heat-transfer measurements; it is probable that these consistent discrepancies are caused by sonically induced microstreaming [29] emanating from minute crevices or discontinuities in the surface of the local heat-transfer test cylinder.

DISCUSSION AND CORRELATION OF RESULTS

In the previous section, it was stated that the mean deviation between the experimental determinations of $(Nu)_o$ and corresponding calculated values from Hilpert's equation (7) is 3 per cent. This close agreement between Hilpert's results and the present investigation is of major importance; for it implies that the intensity of the free stream turbulence encountered throughout the present investigation was close to Hilpert's low value, estimated in [24] to be about 0·9 per cent. Thus, in the large majority of the tests conducted during the present study, the vibrational level, $\epsilon = u/U$, was higher than the intensity of free stream turbulence by an order of magnitude or more. Now, since the intensity of turbulence in the present study was approximately constant and relatively low, and further, since the method used to present the main results of the study [ratios of $(Nu)_v$ to $(Nu)_o$] tends to cancel out the effects of such turbulence, these effects (turbulence effects) upon the main results are considered to be negligible.

During the planning stage of the present work, it was hypothesized that thermoacoustic streaming, which occurs when intense sound is superimposed upon natural convection, would also occur in the case of forced convection, provided that the velocity of the forced convection was sufficiently low. This hypothesis was based on the assumption that for low crossflow velocities, say $U \sim 2$ ft/s [$(Re)_{cf} \sim 1000$], the flow conditions vis-à-vis an intense sound wave are not far different from natural convection. [In this connection, it should be noted that the heat-transfer process for $(Re)_{cf} < 1000$ is, in fact, a combination of natural and forced convection.] The correctness of this anticipated result is supported by the experiments in three ways: (1) Fig. 12, which contains the most extensive set of heat-transfer results obtained in this investigation for particular values of f and $\bar{Q}$, shows that for SPL > 140 dB (140 dB = critical SPL), the lines which connect the data

Table 2. Comparison of average values of h_v, h_o, and $(Nu)_v/(Nu)_o$ for $\bar{Q}$ = 44 W

	Average obtained by integration of local measurements		Average obtained by measuring overall heat transfer	
	h_v or h_o $\left(\frac{\text{Btu}}{\text{h ft}^2\text{ degF}}\right)$	$\frac{h_v}{h_o}$ $\frac{(Nu)_v}{(Nu)_o}$	h_v or h_o $\left(\frac{\text{Btu}}{\text{h ft}^2\text{ degF}}\right)$	$\frac{h_v}{h_o}$ $\frac{(Nu)_v}{(Nu)_o}$
Test A_v SPL = 148 dB f = 1500 cps U = 7·41 ft/s Δt_v = 245·5°F	5·87	1·13	6·12	1·09
Test A_o SPL = 0 U = 7·41 ft/s Δt_o = 268·4°F	5·21		5·63	
Test B_v SPL = 148 dB f = 1500 cps U = 28·3 ft/s Δt_v = 102·7°F	13·9	1·19	14·8	1·14
Test B_o SPL = 0 U = 28·3 ft/s Δt_o = 117·2°F	11·7		13·0	

taken from [13] for natural convection [$(Re)_{cf} = 0$] with the data taken in the present investigation for $(Re)_{cf} \sim 1000$ form a family of continuous, monotonically decreasing curves whose derivatives are also continuous and monotonic. This well-behaved transition from the data at $(Re)_{cf} = 0$ to $(Re)_{cf} \sim 1000$ suggests that the physical process of the interaction between sound and convection throughout this region is the same, namely, thermoacoustic streaming: (2) the local heat-transfer data for $(Re)_{cf} \sim 2000$ in Fig. 17 prove that the increase in heat-transfer due to the injection of sound at low crossflow Reynolds numbers occurs on the upper portion of the cylinder; this effect is similar to the effect of thermoacoustic vortex flow on heat transfer by natural convection from the upper portion of the cylinder in Fig. 1: (3) it was observed visually, by using the smoke injection technique described in [9], that the effect of sound at low crossflow Reynolds numbers was to amplify the vortex motion in the wake; this is characteristic of thermoacoustic streaming. (Motion pictures of the flow were made, but unfortunately these are not of printable quality.)

Fig. 12 shows that the per cent increase in the overall heat-transfer rate caused by a given supercritical field (SPL > 140 dB) is greater in the case of natural convection than for $(Re)_{cf} \sim 1000$; there are two reasons for this difference: first, in the case of forced convection, vortex flow occurs in the absence of sound (as indicated in Test A_o, Fig. 17, by the small loop in the plot of h between 100° and 180°); therefore, in this case the addition of sound acts to amplify an effect which is already present, whereas, in the case of natural convection there is no vortex flow to start with (see cardioid in Fig. 1), and hence the vortices created by sound have a relatively greater effect on the rate of heat

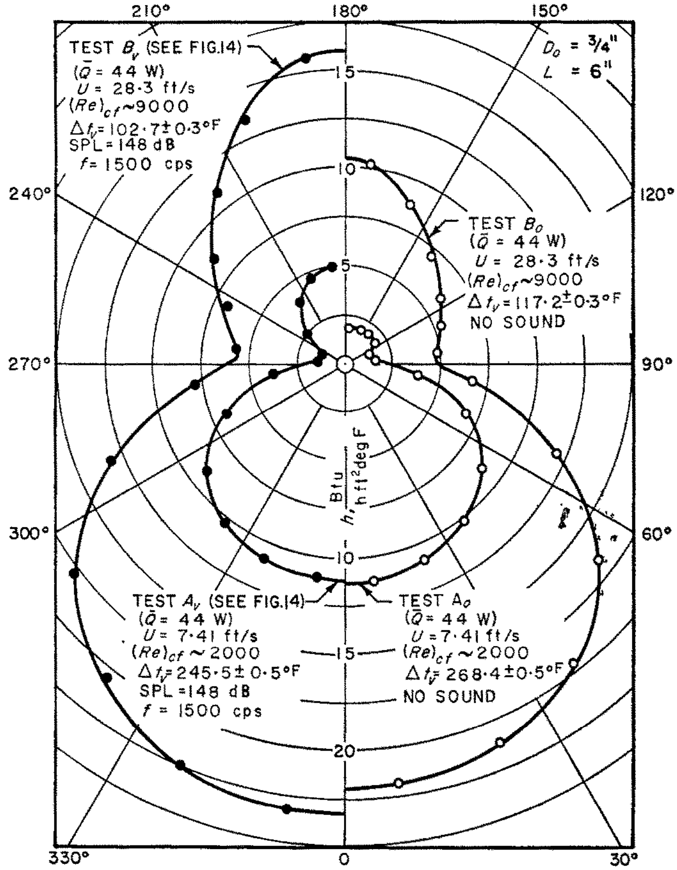


FIG. 17. Local heat transfer coefficients with and without sound for constant $\bar{Q}$ at $(Re)_{cf} \sim 2000$ and $(Re)_{cf} \sim 9000$.

transfer: second, the ratio of the vibrational velocity to the steady component of the velocity in the laminar boundary-layer is much lower for $(Re)_{cf} \sim 1000$ than for natural convection; thus, with crossflow, the vibration represents a weaker disturbance, and its effect on heat transfer from the lower portion of the cylinder is correspondingly less than for the case of natural convection (compare Tests A_o and A_v in Fig. 17 with Fig. 1).

In the region of subcritical sound levels (SPL < 140 dB) and $0 \leqslant (Re)_{cf} < 1000$, Fig. 12 shows an interesting phenomenon: here the ratios $(Nu)_v/(Nu)_o$ possess local maxima near $(Re)_{cf} = 0$, instead of decreasing monotonically as in the case of supercritical sound levels. This behavior is apparently the result of a reduction in the magnitude of the critical SPL when the crossflow Reynolds number is raised from zero (natural convection) to a small finite value.

Figs. 10–16 show that the ratio $(Nu)_v/(Nu)_o$ diminishes almost to 1 as $(Re)_{cf}$ increases from 1000 to approximately 5000; this behavior is to be expected, and the reasons for it are the same as are given above to explain the reduction of the effect of sound on heat transfer as $(Re)_{cf}$ approaches 1000. But this reasoning does not hold throughout the entire experimental range of $(Re)_{cf}$; for Figs. 10–16 show that the ratios $(Nu)_v/(Nu)_o$ are significantly greater than 1 for $(Re)_{cf} \sim 10\,000$. The increase in heat transfer at these higher experimental crossflow Reynolds numbers constitutes the most important result of the present investigation and will now be discussed in detail.

In order to obtain insight into the mechanism whereby sound caused the heat-transfer rate to increase at the higher values of the crossflow Reynolds number [$(Re)_{cf} \sim 10\,000$], local heat-transfer Tests B_0 and B_v in Fig. 17 were performed. The results of these tests show that the injection of sound did not change the position of the point at which separation occurred, nor did it cause transition from laminar to turbulent boundary-layer flow. It follows, then, that for $(Re)_{cf} \sim 10\,000$ the sound field augmented the heat-transfer rate by modifying the laminar boundary layer (0–100°) and by changing the flow in the wake (100–180°). The mechanism of interaction in the region of the laminar boundary layer is very likely the same as is encountered in the case of thermoacoustic streaming and free stream turbulence (see Survey of the Literature); but, for $(Re)_{cf} \sim 10\,000$, the interaction in the region of the wake cannot be the same as occurs in thermoacoustic streaming, because it has already been shown that the thermoacoustic mechanism becomes ineffective when $(Re)_{cf}$ reaches approximately 5000. The question therefore arises: what is the mechanism of interaction in the wake for $(Re)_{cf} \sim 10\,000$? A plausible answer appears to be that the mechanism in question is resonance between the acoustic oscillation and the vortices which are normally shed from the cylinder. This kind of resonance has been proposed by Van der Hegge Zijnen to explain certain of his observations concerning the effect of free stream turbulence on heat transfer (see Survey of the Literature). The experimental evidence which suggests the resonance hypothesis in the present work is the fact that the ratio $(Nu)_v/(Nu)_o$ begins to rise for $(Re)_{cf}$ approximately equal to 4500 when $f = 1100$ cps (Figs. 10 and 11) and for $(Re)_{cf}$ approximately equal to 6000 when $f = 1500$ cps (Figs. 12–16). Now, since $(Re)_{cf}$ is proportional to the crossflow velocity, and therefore to the Strouhal frequency (the frequency of vortex shedding), and since the ratio of these Reynolds numbers ($6000/4500 = 1{\cdot}33$) is almost exactly equal to the ratio of the corresponding frequencies ($1500/1100 = 1{\cdot}36$), the resonance hypothesis is strongly suggested. It should be noted that the preceding argument utilizes the values of $(Re)_{cf}$ at which $(Nu)_v/(Nu)_o$ *begins* to increase; clearly, resonance does not occur at these values of $(Re)_{cf}$. These values of $(Re)_{cf}$ are assumed here to be proportional to the values at which resonance actually does occur; this assumption was made for lack of more direct measurements in the wake. In this connection it might be worthwhile to mention that the values of $(Re)_{cf}$ at which resonance occurs are not equal to $(Re)^*_{cf}$, because $(Re)^*_{cf}$ depends upon the influence of sound on the heat-transfer rate to the laminar boundary layer as well as to the wake.

It has been pointed out that the ratios $(Nu)_v/(Nu)_o$ pass through minimum values when $(Re)_{cf} \simeq 4500$ for $f = 1100$ cps and when $(Re)_{cf} \simeq 6000$ for $f = 1500$ cps; however, these minimum values are not constant, they depend upon the temperature difference. This is clearly demonstrated by Figs. 12–16 which show that the minimum values of $(Nu)_v/(Nu)_o$ increase from 1·01 to about 1·05 with an increase in $\bar{Q}$ from 20 to 80 W, i.e. with an increase in Δt_v from 75° to 250°F. The reason for this observed increase in the minimum values of $(Nu)_v/(Nu)_o$ with Δt_v has not been determined at this writing.

The numerical values of $(Nu)^*$ are of special interest, because they represent the maximum effectiveness of sound as an agent for increasing the rate of heat transfer from a cylinder in a useful range of $(Re)_{cf}$; for this reason, an empirical method has been devised for calculating $(Nu)^*$. The method is similar to Van der Hegge Zijnen's analysis of the effect of free stream turbulence on heat transfer and begins with an equation analogues to equation (1), namely

$$(Nu)^* = 1 + [\phi\,(Re)^*_v]\left[\psi\left(\frac{L_x}{D_o}\right)\right] \tag{8}$$

where ϕ is a function of the level of vibration, expressed in terms of the vibrational Reynolds number, $(Re)^*_v = \epsilon(Re)^*_{cf}$; and ψ is a function of (L_x/D_o), where L_x, the scale of the sound field, is defined as follows:

L_x = distance a particle moves in the direction of crossflow during a half cycle of vibration.

Defined in this way, L_x is the distance the particle moves in the direction of crossflow during the time it leaves its equilibrium position and returns. Thus, $L_x = U/2f$, from which $L_x/D_o = U/2fD_o$. The function ψ used in the present analysis was obtained by approximating the first part of Van der Hegge Zijnen's curve shown in Fig. 6 by a straight line segment. The equation of this line (shown dotted in Fig. 6) is

$$\psi = 0{\cdot}016\left(\frac{L_x}{D_o}\right). \qquad (9)$$

In order to obtain an expression for $\phi(Re)^*_v$, it was first necessary to devise a method for calculating $(Re)^*_{cf}$ for a given sound field and temperature difference. This was done by plotting all the experimentally determined values of $(Re)^*_{cf}$ in the manner shown in Fig. 18. A numerical analysis of this plot reveals that all these data can be correlated by the following empirical equation:

$$(Re)^*_{cf} = -37\,(\mathrm{SPL}) + \frac{b}{\delta_{ac}\omega^{1/4}} \qquad (10)$$

where $b = 21{\cdot}1 \pm 0{\cdot}2$. The sound pressure level in (10) can be replaced by the following expression involving u:

$$\mathrm{SPL} = 20 \log u + 136. \qquad (11)$$

Equation (10) provides a means for calculating $(Re)^*_{cf}$; therefore, since $(Re)^*_v = \epsilon(Re)^*_{cf}$, the analysis can be completed by determining the dependence of $(Nu)^*$ on ϵ.

In order to obtain a clue to the functional dependence of $(Nu)^*$ upon the level of vibration, ϵ, the values of $[(Nu)_v/(Nu)_o - 1]$ in Fig. 10 for $(Re)_{cf} = 9000$ were crossplotted as a function of ϵ; this procedure was helpful because for $(Re)_{cf} = 9000$, the values of $(Nu)_v/(Nu)_o$ are negligibly different from $(Nu)^*$. The crossplot showed that the increment in heat transfer due to sound, $[(Nu)_v/(Nu)_o - 1]$, is nearly proportional to the cube root of ϵ. This clue was utilized in conjunction with the experimental data in Figs. 10–16 to obtain the following result:

$$\phi = 5{\cdot}30\,[(Re)^*_v]^{1/3}. \qquad (12)$$

A plot of (12) is shown in Fig. 5; the curve resembles Van der Hegge Zijnen's analogous result for turbulence.

The determination of (12) completes the analysis, because for a given cylinder, given sound field, and given temperature difference ($D_o, u, f, \Delta t$) it is now possible to compute $(Nu)^*$.

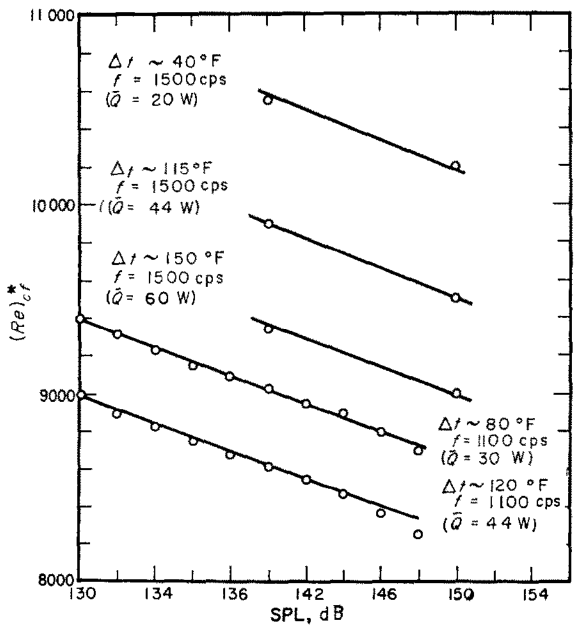


FIG. 18. $(Re)_{cf}$* versus SPL.

*Table 3. Comparison of calculated and measured values of (Nu)**

SPL (dB)	$(Nu)_{meas.}^*$	$(Re)_{cf}^*$	U (ft/s)	u (ft/s)	$(Re)_v^* = \frac{u}{U}(Re)_{cf}^*$	$\phi = 5{\cdot}30\,[(Re)_v^*]^{1{\cdot}3}$	$\frac{L_x}{D_o} = \frac{U}{2f\,D_o}$	$\psi = 0{\cdot}016\left(\frac{L_x}{D_o}\right)$	$(Nu)_{calc.}^* = 1 + (\phi)(\psi)$	$\frac{(Nu)_{meas.}^* - (Nu)_{calc.}^*}{(Nu)_{meas.}^*}$ 100
f = 1100 cps; $\bar{Q}$ = 30 W; $t_f \cong$ 102°F; ν_f = 18·4 × 10^{-5} ft²/s										
148	1·191	8680	25·6	4·01	1360	58·7	0·186	0·00297	1·175	1·3
140	1·133	8970	26·4	1·59	540	43·2	0·192	0·00307	1·133	0·0
130	1·088	9340	27·5	0·498	169·5	29·4	0·200	0·00320	1·094	−0·6
f = 1100 cps; $\bar{Q}$ = 44 W; $t_f \cong$ 123°F; ν_f = 19·3 × 10^{-5} ft²/s										
148	1·200	8310	25·5	4·01	1306	58·0	0·185	0·00297	1·172	2·4
140	1·143	8600	26·6	1·59	514	42·5	0·194	0·00311	1·132	0·9
130	1·085	8970	27·7	0·498	161·5	28·9	0·202	0·00323	1·094	−0·8
f = 1500 cps; $\bar{Q}$ = 20 W; $t_f \cong$ 90°F; ν_f = 17·5 × 10^{-5} ft²/s										
150	1·151	10170	28·4	5·05	1808	64·5	0·154	0·00246	1·159	−0·7
140	1·110	10540	29·6	1·59	566	43·8	0·159	0·00256	1·112	−0·2
130	1·076	10910	30·6	0·498	178·0	29·8	0·163	0·00262	1·078	−0·2
f = 1500 cps; $\bar{Q}$ = 44 W; $t_f \cong$ 116°F; ν_f = 18·9 × 10^{-5} ft²/s										
150	1·162	9550	29·0	5·05	1661	62·8	0·154	0·00247	1·155	0·6
140	1·102	9920	30·0	1·59	526	42·7	0·160	0·00256	1·092	0·9
f = 1500 cps; $\bar{Q}$ = 60 W; $t_f \cong$ 140°F; ν_f = 20·2 × 10^{-5} ft²/s										
150	1·152	9070	29·3	5·05	1561	61·5	0·156	0·00249	1·153	−0·1
140	1·100	9440	30·5	1·59	492	41·8	0·161	0·00258	1·082	−1·6
D_o = 0·0625 ft									Mean Deviation, per cent	0·8

The calculation can be performed as follows: first, the value of $(Re)^*_{cf}$ is obtained from (10) and (11); then, the crossflow velocity corresponding to $(Re)^*_{cf}$ is computed; next, the appropriate values of ϕ and ψ are obtained from the definitions of $(Re)^*_v$ and L_x and by utilizing (9) and (12); finally, these values of ϕ and ψ are inserted into (8) to obtain $(Nu)^*$. In the preceding calculation, the given film temperature may be used to evaluate the kinematic viscosity which appears in $(Re)^*_{cf}$ with little loss of accuracy; however, if desired, it is possible to eliminate this slight error by iteration. Table 3 contains a comparison between thirteen representative measured values of $(Nu)^*$ and corresponding values calculated from (8); the mean deviation between the two sets of values is 0·8 per cent, which indicates good agreement.

RÉSUMÉ AND CONCLUSIONS

The effect of sound on the rate of heat transfer from a cylinder in crossflow has been shown to be a complicated function of the sound frequency and intensity, the temperature difference, and the crossflow velocity. No general theoretical or empirical method for predicting the magnitude of this effect has yet been devised. Kubanskii's equation (5) represents an attempt to achieve a correlation of this kind, but this equation is based on data limited to $(Re)_{cf}$ from 1450 to 1770, and it does not account for the rise in the rate of heat transfer which was observed to occur in the present study when $(Re)_{cf} > 6000$; nor does it account for changes in the heat-transfer rate caused by changes in f or Δt. Furthermore, on the basis of the present as well as previous studies [10], it appears that the basic assumption which Kubanskii made in his theoretical analysis of this problem, namely, that the simple superposition of acoustic streaming on crossflow can explain the influence of sound on heat transfer, is not valid. The interaction is nonlinear, and for nonlinear interactions the method of simple superposition is not applicable.

The results of this investigation show that intense sound causes the overall convective heat-transfer coefficient to increase appreciably in two regions. In one of these regions, which corresponds to the lowest values of the crossflow Reynolds number employed in these experiments [$(Re)_{cf} \sim 1000$], the increase in the rate of heat transfer appears to be the result of an interaction similar to thermoacoustic streaming. In the second region, which corresponds to the highest values of the crossflow Reynolds number employed in the experiments [$(Re)_{cf} \sim 10\,000$], the increase in heat transfer appears to be the result of two different interactions: (1) a resonance interaction between the acoustic oscillations and the vortices shed from the cylinder; (2) a modification of the flow in the laminar boundary layer on the upstream portion of the cylinder similar to the effect of free stream turbulence. The only direct experimental evidence which has been obtained in this study in support of the resonance hypothesis is the observation that the values of $(Re)_{cf}$ at which $(Nu)_v/(Nu)_o$ begins to rise are proportional to f. Although this evidence is highly suggestive, it cannot be regarded as conclusive; conclusive evidence can only be obtained by direct measurements in the wake. For this reason it is anticipated that further investigations of these phenomena will require the use of hot wire anemometry.

In the region where it is suggested that resonance occurs, the maximum effect of sound on the overall rate of heat transfer can be computed from the following formula:

$$(Nu)^* = 1 + (\phi)(\psi). \tag{8}$$

This formula has been used by Van der Hegge Zijnen [27] to correlate the effects of free stream turbulence on heat transfer. With the aid of equations (9) and (12), (8) can be written as follows:

$$(Nu)^* = 1 + 0{\cdot}848\,[(Re)^*_v]^{1/3}\left(\frac{L_x}{D_o}\right) \tag{13}$$

where $(Re)^*_v = \epsilon(Re)^*_{cf}$

and

$$(Re)^*_{cf} = -37\,\mathrm{SPL} + \frac{21{\cdot}1}{\delta_{ac}\omega^{1/4}}. \tag{10}$$

The quantitative result embodied in (13) is of interest for two reasons; first, because the equation provides a means for predicting $(Nu)^*$; second, and more important, because the equation indicates that there is an analogy between the effect of sound and the effect of free stream turbulence on heat transfer. The existence of an analogy is indicated by the similarity in form of equations (6) and (8) and by the similarity of the pairs of functions in Figs. 5 and 6. This analogy is important because it implies that common measures of sound and turbulence exist, and hence certain properties and effects of turbulence can be studied by using acoustic vibrations as a tool; such a procedure would offer distinct advantages, because it is much easier to create and control acoustic fields having different intensities and frequencies than it is to control the parameters of turbulence, and also, because an acoustic (harmonic) field is easier to treat mathematically than a turbulent field.

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Résumé—Cet article rapporte les résultats d'une étude expérimentale concernant l'influence des vibrations acoustiques intenses sur le taux de transmission de chaleur d'un cylindre circulaire (diamètre 2 cm) placé normalement à un écoulement d'air.

La direction de la vibration est à la fois normale à l'axe du cylindre et à l'écoulement. On a utilisé des ondes sonores stationnaires planes et le cylindre a été placé de façon que son axe longitudinal coïncide avec le déplacement des ventres des ondes; les fréquences des vibrations utilisées étaient de 1100 à 1500 c/s.

Les résultats montrent que les vibrations sonores provoquent un accroissement appréciable du coefficient de convection moyen (allant jusqu'à 25%) dans deux domaines. Dans le premier, qui correspond aux valeurs les plus basses du nombre de Reynolds de l'écoulement transversal utilisées dans les expériences $[(Re)_{cf} \sim 1000]$, l'accroissement du coefficient d'échange thermique semble être le résultat d'une interaction semblable aux écoulements développés par les phénomènes thermo-acoustiques. Dans le second domaine, qui correspond aux valeurs les plus élevées du nombre de Reynolds $[(Re)_{cf} \sim 10.000]$ l'accroissement du taux d'échange semble être dû à deux interactions différentes (1) l'une provoquée par la résonance entre les oscillations acoustiques et les tourbillons engendrés par le cylindre (2) la modification de l'écoulement dans la couche limite sur la partie amont du cylindre identique à l'effet que produit la turbulence de l'écoulement libre. Les mesures locales de transmission de chaleur confirment cette hypothèse. Des équations empiriques permettant le calcul de l'accroissement maximum du nombre de Nusselt dû à une onde sonore déterminée dans le second domaine $[(Re_{cf} \sim 10.000)]$ sont développées.

Zusammenfassung—Es wird über den experimentell untersuchten Einfluss intensiver akustischer Schwingungen auf den Wärmeübergang an einem von Luft quer angeströmten Kreiszylinder (19 mm Durchmesser) berichtet. Die Schwingrichtung lag senkrecht zu Zylinderachse und Strömungsrichtung. Der Zylinder war so angeordnet, dass seine Längsachse mit den Schwingungsbäuchen ebener stationärer Schallwellen zusammenfiel; die Schwingfrequenzen betrugen 1100 Hz und 1500 Hz. Die Ergebnisse zeigen, dass intensiver Schall zu einer merklichen (bis zu 25 Prozent) Vergrösserung des konvektiven Gesamtwärmeübergangskoeffizienten in zwei Bereichen führt. Im einen Bereich, dem der kleinsten für die Strömung ermittelten Reynoldszahlen $[(Re)_{cf} \sim 1000]$ scheint die Erhöhung des Wärmeübergangs auf einer Wechselwirkung ähnlich der thermostatischen Strömung zu beruhen. Im zweiten Bereich, in dem die Reynoldszahlen die grösstmöglichen Versuchswerte annehmen $[(Re)_{cf} \sim 10\,000]$ wird die Vergrösserung auf zwei verschiedene Einflüsse zurückgeführt: (1) auf eine Resonanzwirkung zwischen den akustischen Schwingungen und den vom Zylinder abgelösten Wirbeln; (2) auf eine Veränderung der laminaren Grenzschichtströmung im vorderen, der Strömung zugewandten Teil des Zylinders, ähnlich dem Einfluss der Freistromturbulenz. Die angegebenen, örtlichen Wärmeübergangswerte unterstützen diese Annahme. Empirische Gleichungen wurden entwickelt; sie gestatten den von einer bestimmten Schallwelle im zweiten Bereich $[(Re)_{cf} \sim 10\,000]$ verursachten Maximalanstieg der Nusseltzahl zu berechnen.

Аннотация—В статье приводятся результаты экспериментального исследования влияния сильных акустических колебаний на интенсивность теплообмена круглого цилиндра (диаметром $\frac{3}{4}$ дюйма) при поперечном обтекании потоком воздуха. Направление колебаний было перпендикулярным к оси цилиндра и направлению течения. Создавались плоские стационарные звуковые волны с частотами 1100 периодов/сек и

1500 периодов/сек. Продольная ось цилиндра совпадала с направлением пучностей звуковых волн.

В результате опытов найдено, что сильные звуковые колебания заметно увеличивают общий коэффициент конвективного теплообмена (до 25%) в двух областях. В одной их них, соответствующей минимальным числам Рейнольдса в данном эксперименте $[(Re)_{cf} \sim 1000]$, интенсификация теплообмена вызывается воздействием, аналогичным термостатическому потоку. В другой области, соответствующей максимальным значением чисел Рейнольдса в опыте $[(Re)_{cf} \sim 10000]$ интенсификация теплообмена происходит в результате: (1) резонансного взаимодействия между акустическими колебаниями и завихрениями от цилиндра и (2) перестройки течения в ламинарном пограничном слое в передней части цилиндра, подобно тому, как это происходит в случае турбулизации свободного потока. Приводятся данные по локальному теплообмену, подтверждающие эту гипотезу. Полчены эмпирические уравнения с помощью которых можно рассчитать максимальное приращение числа Нуссельта, связанное с акустическим эффектом во второй области $[(Re)_{cf} \sim 10000]$.